

Initial state radiation to off-shell Z^0 pair production in e^+e^- annihilation*

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Abstract

A study of the Standard Model reaction $e^+e^- \rightarrow (Z^0Z^0) \rightarrow f_1\bar{f}_1f_2\bar{f}_2$ including the effects of the finite Z^0 width and initial state radiative corrections is presented. All angular phase space integrations are performed analytically. The remaining invariant masses are integrated numerically. Semi-analytical and numerical results in the energy range $\sqrt{s} = 200 \text{ GeV}$ to 1 TeV are reported.

1 Introduction

For LEP 2 and a planned 500 GeV e^+e^- collider, annihilation into boson pairs is a major issue, because double resonance production is strongly enhanced. LEP 2 will operate above the W^+W^- pair production threshold, perhaps above the Z^0Z^0 threshold and will open a new discovery window for a Higgs boson in the Z^0H channel. Since all heavy bosons decay, their finite widths must be taken into account. Furthermore radiative corrections are needed to match theoretical with experimental precision. Thus, numerous efforts were made to theoretically describe boson pair production and four fermion final states. In this paper I restrict myself to the unpolarized reaction

$$e^+e^- \rightarrow (Z^0Z^0) \rightarrow f_1\bar{f}_1f_2\bar{f}_2, \quad f_1 \neq f_2, \quad f_i \neq e. \quad (1)$$

including the effects of the finite Z^0 width and QED Initial State Radiation (ISR). The motivation for this is fourfold. Process (1) should be measurable at LEP 2 energies and above. It represents a genuine higher order test of the electroweak Standard Model, and, for some channels, it yields a background for W^+W^- physics as well as Higgs boson searches. In addition, this study can be easily extended to nonstandard neutral current physics, e.g. Z' bosons exchanges.

On-shell Z^0 pair production has been discussed long ago [1]. Numerical calculations including all $\mathcal{O}(\alpha)$ electroweak corrections except hard photon bremsstrahlung were reported in [2] for on-shell and in [3] for off-shell Z^0 bosons. Monte Carlo generators including

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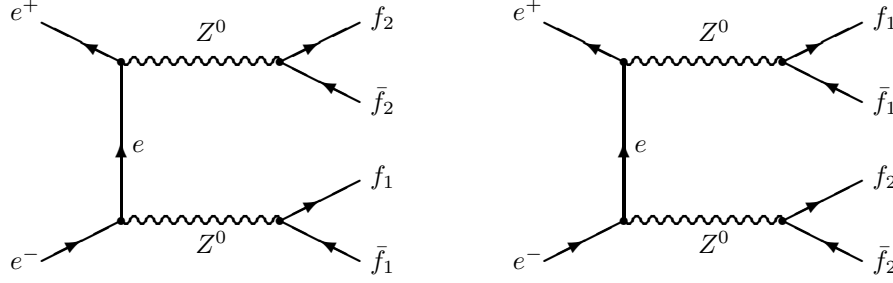


Figure 1: *The Born level Feynman diagrams for off-shell Z^0 pair production. Left: t -channel. Right: u -channel.*

radiative corrections were described in [4, 5]. On-shell calculations fail to take the important finite width effects into account. Furthermore ISR is treated incompletely in references [2, 3]. References [3, 4, 5] strongly rely on numerical phase space integration. In this note, in an effort to augment the understanding of process (1), off-shell complete ISR is calculated following a semi-analytical approach. This means that all angular phase space variables are integrated analytically, leaving three invariant masses for numerical integration. Quasi-experimental cuts on these can be easily implemented.

The paper is organized as follows. In section 2 semi-analytical results for the off-shell Born cross-section are presented, followed by the ISR results in section 3. Section 4 contains a short discussion of backgrounds to process (1) and a few remarks on the results' gauge behaviour. Section 5 closes the paper with a summary, an outlook and conclusions.

2 The Born Cross-Section

At Born level, process (1) is described by the two Feynman diagrams depicted in figure 1. The Born cross-section is given by a simple convolution formula

$$\sigma^{ZZ}(s) = \int_{4m_1^2}^s ds_1 \rho_Z(s_1) \int_{4m_2^2}^{(\sqrt{s}-\sqrt{s_1})^2} ds_2 \rho_Z(s_2) \cdot \sigma_4^{ZZ}(s; s_1, s_2) \quad (2)$$

invoking Breit-Wigner density functions for the s-channel Z^0 propagators:

$$\rho_Z(s_i) = \frac{1}{\pi} \frac{\sqrt{s_i} \Gamma_Z(s_i) \times BR(i)}{|s_i - M_Z^2 + i\sqrt{s_i} \Gamma_Z(s_i)|^2}. \quad (3)$$

$BR(i)$ is the branching ratio for the decay channel under study, s_1 and s_2 are the invariant Z^0 masses. The Z^0 width is given by

$$\Gamma_Z(s_i) = \frac{G_\mu M_Z^2}{24\pi\sqrt{2}} \sqrt{s_i} \sum_f (v_f^2 + a_f^2). \quad (4)$$

Using $a_e=1$, $v_e=1-4\sin^2\theta_W$, $L_e=(a_e+v_e)/2$ and $R_e=(a_e-v_e)/2$, $\sigma_4^{ZZ}(s; s_1, s_2)$ is obtained after fivefold analytical integration over the angular phase space variables.

$$\sigma_4^{ZZ}(s; s_1, s_2) = \frac{(G_\mu M_Z^2)^2}{8\pi s} (L_e^4 + R_e^4) \mathcal{G}_4^{t+u}(s; s_1, s_2). \quad (5)$$

The sub-index 4 indicates that the underlying matrix element contains four resonant propagators. Although \mathcal{G}_4^{t+u} is the sum of three kinematical functions stemming from the t-channel, the u-channel, and the t-u interference, it can be very compactly written as:

$$\mathcal{G}_4^{t+u}(s; s_1, s_2) = \frac{\lambda^{1/2}}{s} \left[\frac{s^2 + (s_1 + s_2)^2}{s - s_1 - s_2} \mathcal{L}_4 - 2 \right] \quad (6)$$

with $\lambda \equiv s^2 + s_1^2 + s_2^2 - 2ss_1 - 2s_1s_2 - 2s_2s$ and

$$\mathcal{L}_4(s; s_1, s_2) = \frac{1}{\sqrt{\lambda}} \ln \frac{s - s_1 - s_2 + \sqrt{\lambda}}{s - s_1 - s_2 - \sqrt{\lambda}}. \quad (7)$$

This result was only recently derived [6]. In the on-shell limit, eq. (3) yields $\rho_Z(s_i) = \delta(s_i - M_Z) \cdot BR(i)$ and, with $\beta_Z = \sqrt{1 - 4M_Z^2/s}$, eq. (6) becomes

$$\mathcal{G}_4^{t+u}(s; M_Z, M_Z) = 2 \cdot \left[\frac{1 + 4M_Z^2/s}{1 - 2M_Z^2/s} \ln \frac{1 + \beta_Z}{1 - \beta_Z} - \beta_Z \right], \quad (8)$$

being in agreement with [1]. The effect of the finite Z^0 width can be seen from figure 2 as the characteristic smearing of the peak.

Figure 2: *The total cross-section $\sigma^{ZZ}(s)$ for process (1).*

3 $\mathcal{O}(\alpha)$ Initial State Radiation

In e^+e^- annihilation, ISR is known to represent the bulk of the radiative corrections. The $\mathcal{O}(\alpha)$ ‘amputated’ Feynman diagrams for Initial State Bremsstrahlung (ISB) to process (1) are depicted in figure 3. The corresponding virtual ISR diagrams are shown in figure 4. External leg self energies are absorbed into the on-shell renormalization. The double-differential cross-section for off-shell Z^0 pair production including $\mathcal{O}(\alpha)$ ISR with soft photon exponentiation can be presented as

$$\frac{d^2\sigma^{ZZ}}{ds_1 ds_2} = \int_{(\sqrt{s_1}+\sqrt{s_2})^2}^s \frac{ds'}{s} \rho(s_1) \rho(s_2) \left[\beta_e v^{\beta_e-1} \mathcal{S} + \mathcal{H} \right] \quad (9)$$

with $\beta_e = 2\alpha/\pi[\ln(s/m_e^2) - 1]$ and $v = 1 - s'/s$. The soft+virtual and hard photonic parts \mathcal{S} and \mathcal{H} are calculated analytically, requiring seven angular integrations. Both separate into a universal part with the Born cross-section factorizing and a nonuniversal part:

$$\begin{aligned} \mathcal{S}(s, s'; s_1, s_2) &= [1 + \bar{S}(s)] \sigma_0(s'; s_1, s_2) + \sigma_{\bar{S}}(s'; s_1, s_2) \quad , \\ \mathcal{H}(s, s'; s_1, s_2) &= \underbrace{\bar{H}(s, s') \sigma_0(s'; s_1, s_2)}_{\text{Universal Part}} + \underbrace{\sigma_{\bar{H}}(s, s'; s_1, s_2)}_{\text{Nonuniversal Part}} \quad . \end{aligned} \quad (10)$$

An explicit derivation proved that as expected \bar{S} and \bar{H} are identical to the radiators known from s-channel fermion pair production [7]:

$$\begin{aligned} \bar{S}(s) &= \frac{\alpha}{\pi} \left[\frac{\pi^2}{3} - \frac{1}{2} \right] + \frac{3}{4} \beta_e + \mathcal{O}(\alpha^2), \\ \bar{H}(s, s') &= -\frac{1}{2} \left(1 + \frac{s'}{s} \right) \beta_e + \mathcal{O}(\alpha^2). \end{aligned} \quad (11)$$

The analytical calculation of the nonuniversal contributions is under way, its bremsstrahlung part already completed. Since the corresponding analytical expressions contain many Spence functions, nonuniversal contributions are involved. On the other hand they are small, because they do not contain the mass singularity β_e . Similar arguments hold for W^+W^- pair production [6, 8]. The off-shell cross-section of process (1), corrected for universal ISR, is presented in figure 2. It is radiatively reduced below the peak and develops a strong radiative tail above.

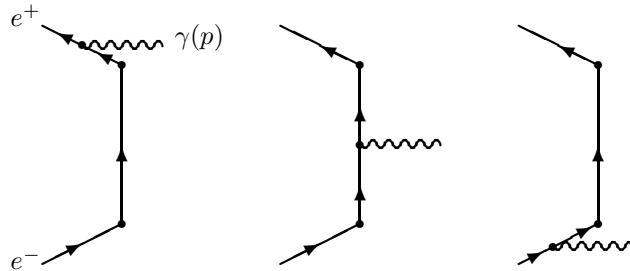


Figure 3: *The amputated ISB diagrams for Z^0 pair production.*

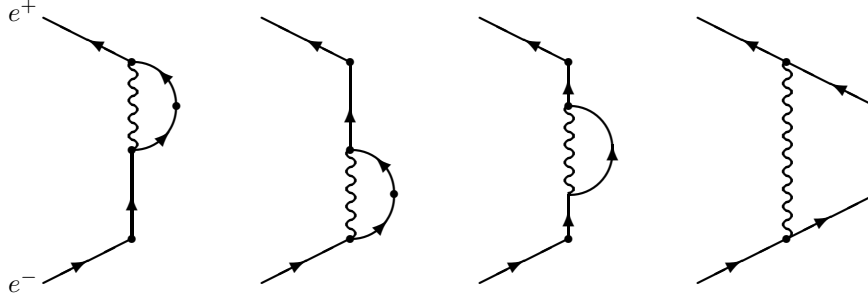


Figure 4: *The amputated virtual ISR diagrams for Z^0 pair production.*

4 Background

Strictly speaking, gauge invariance requires that all Feynman diagrams contributing to $e^+e^- \rightarrow f_1\bar{f}_1f_2\bar{f}_2$ be taken into account. This means the inclusion of not only singly and nonresonant diagrams as given in figure 2 of [6], but also diagrams with photons replacing the Z^0 bosons and, for some final states, charged current diagrams. However, as they are suppressed by a factor Γ_Z/M_Z for each nonresonating boson propagator, singly and nonresonant diagrams only yield small contributions [5, 9]. Their smallness for the case of W^+W^- pair production was proven in [5, 6]. Photon exchange diagrams can also be considered as background, if, similar to experimental situations, invariant mass cuts are applied to isolate decaying Z^0 bosons. Such cuts essentially extinguish photon exchange contributions, but have a small effect on resonant diagrams as can be seen from figure 5. Slight gauge violations also come with the introduction of finite boson widths. A scheme to avoid these was proposed, but gives incorrect results around threshold [10]. Up to now, no scheme for the introduction of finite widths into boson pair processes seems theoretically satisfactory. However, different schemes only differ in higher order.

5 Summary, Outlook and Conclusions

I have reported finite width and initial state QED corrections to $e^+e^- \rightarrow (Z^0Z^0) \rightarrow f_1\bar{f}_1f_2\bar{f}_2$ in a semi-analytical approach. It was shown that both yield important corrections to the total cross-section. Final State Radiation (FSR) and Initial-Final Interferences (IFI) become important at very high energies. FSR can be implemented in a radiator approach, but it is unclear, if the semi-analytical treatment is efficient for IFI.

Next, the calculation of nonuniversal contributions to ISR has to be completed. After this the inclusion of photon exchange graphs is natural. For the future it is planned to compute singly and nonresonant background, its contribution to the cross-section being of comparable magnitude as nonuniversal ISR. Further plans comprise FSR and IFI calculations. For the more remote future, the inclusion of weak corrections and non-standard neutral current physics are likely extensions of the presented work.

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Figure 5: *Effect of a cut $|s_i - M_Z| \leq 10 \text{ GeV}$ on the off-shell Born cross-section.*

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